| Student Name: | BJUT ID: | UCD ID: |
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Institution: Beijing-Dublin International College
Module: University Physics 2 (BDIC2008J)
Lecturer: Dr. Hao Zhu

# Problem Set 7

*Electromagnetic Induction*

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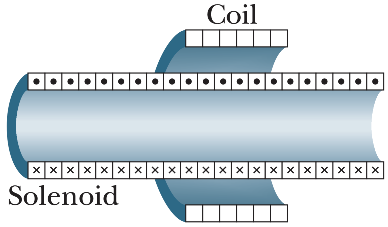
**Problem 1.** *In the Figure below, a* 120-turn *coil of radius* $1.8\,\mathrm{cm}$ *and resistance* $5.3\,\Omega$ *is coaxial with a solenoid of* $220\,\mathrm{turns/cm}$ *and diameter* $3.2\,\mathrm{cm}$*. The solenoid current drops from* $1.5\,\mathrm{A}$ *to zero in the time interval* $\Delta t = 25\,\mathrm{ms}$*. What current is induced in the coil during* $\Delta t$*?*




*Solution.* Changing the current in the solenoid changes the flux, and therefore, induces a current in the coil. Using Faraday's law, the total induced emf is given by

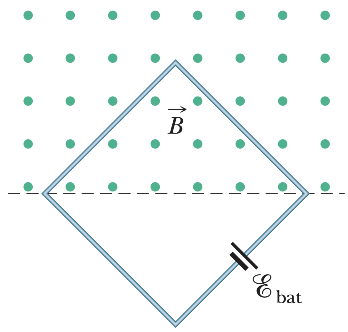
$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NA\left(\frac{dB}{dt}\right) = -NA\frac{d}{dt}(\mu_0 n i) = -N\mu_0 n A\frac{di}{dt} = -N\mu_0 n(\pi r^2)\frac{di}{dt}$$

By Ohm's law, the induced current in the coil is $i_{\text{ind}} = |\varepsilon|/R$, where $R$ is the resistance of the coil. Substituting the values given, we obtain

$$\begin{aligned}\varepsilon &= -N\mu_0 n(\pi r^2)\frac{di}{dt} = -(120)(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(22000\,/\mathrm{m})\pi(0.016\,\mathrm{m})^2\left(\frac{1.5\,\mathrm{A}}{0.025\,\mathrm{s}}\right)\\ &= 0.16\,\mathrm{V}\end{aligned}$$

Ohm's law then yields $i_{\text{ind}} = \dfrac{|\varepsilon|}{R} = \dfrac{0.016\,\mathrm{V}}{5.3\,\Omega} = 0.030\,\mathrm{A}$. □

**Problem 2.** *A square wire loop with* $2.00\,\mathrm{m}$ *sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in the Figure. The loop contains an ideal battery with emf* $\mathscr{E} = 20.0\,\mathrm{V}$. *If the magnitude of the field varies with time according to* $B = 0.0420 - 0.870\,t$, *with* $B$ *in teslas and* $t$ *in seconds, what are* ***(a)*** *the net emf in the circuit and* ***(b)*** *the direction of the (net) current around the loop?*




*Solution.* **(a)** Let $L$ be the length of a side of the square circuit. Then the magnetic flux through the circuit is $\Phi_B = L^2 B/2$, and the induced emf is

$$\varepsilon_i = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt}$$

Now $B = 0.042 - 0.870\,t$ and $dB/dt = -0.870\,\mathrm{T/s}$. Thus,

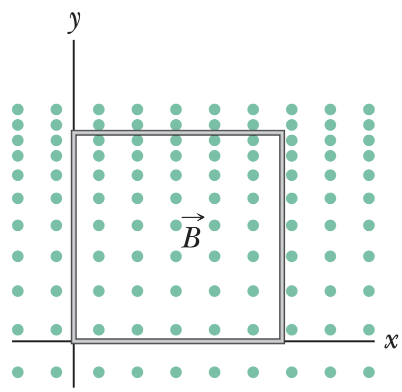
$$\varepsilon_i = \frac{(2.00\,\mathrm{m})^2}{2}(0.870\,\mathrm{T/s}) = 1.74\,\mathrm{V}$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is

$$\varepsilon + \varepsilon_i = 20.0\,\mathrm{V} + 1.74\,\mathrm{V} = 21.74\,\mathrm{V}$$

**(b)** The current is counterclockwise. □

**Problem 3.** *As seen in the Figure, a square loop of wire has sides of length* 2.0 cm*. A magnetic field is directed out of the page; its magnitude is given by* $B = 4.0t^2y$*, where* $B$ *is in teslas,* $t$ *is in seconds, and* $y$ *is in meters. At* $t = 2.5\,\mathrm{s}$*, what are the* ***(a)*** *magnitude and* ***(b)*** *direction of the emf induced in the loop?*




*Solution.* **(a)** Consider a (thin) strip of area of height $dy$ and width $l = 0.020\,\mathrm{m}$. The strip is located at some $0 < y < l$. The element of flux through the strip is

$$d\Phi_B = B\,dA = (4t^2y)(l\,dy)$$

where SI units (and 2 significant figures) are understood. To find the total flux through the square loop, we integrate:

$$\Phi_B = \int d\Phi_B = \int_0^l (4t^2yl)\,dy = 2t^2l^3$$

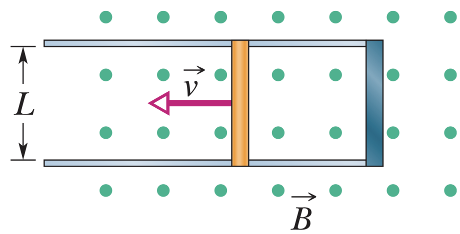
Thus, Faraday's law yields

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = 4tl^3$$

At $t = 2.5\,\mathrm{s}$, the magnitude of the induced emf is $8.0 \times 10^{-5}\,\mathrm{V}$.

**(b)** Its "direction" is clockwise, by Lenz's law. □

**Problem 4.** *In the Figure below, a metal rod is forced to move with constant velocity $\vec{v}$ along two parallel metal rails, connected with a strip of metal at one end. A magnetic field of magnitude $B = 0.350\,\mathrm{T}$ points out of the page.* ***(a)*** *If the rails are separated by $L = 25.0\,\mathrm{cm}$ and the speed of the rod is $55.0\,\mathrm{cm/s}$, what emf is generated?* ***(b)*** *If the rod has a resistance of $18.0\,\Omega$ and the rails and connector have negligible resistance, what is the current in the rod?* ***(c)*** *At what rate is energy being transferred to thermal energy?*




*Solution.* **(a)** The equation of motive emf leads to

$$\varepsilon = BLv = (0.350\,\mathrm{T})(0.250\,\mathrm{m})(0.55\,\mathrm{m/s}) = 0.0481\,\mathrm{V}$$

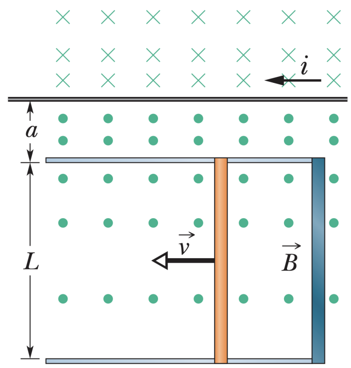
**(b)** By Ohm's law, the induced current is

$$i = 0.0481\,\mathrm{V}/18.0\,\Omega = 0.00267\,\mathrm{A}$$

By Lenz's law, the current is clockwise.

**(c)** The power leads to $P = i^2 R = 0.000129\,\mathrm{W}$. □

**Problem 5.** *The figure below shows a rod of length $L = 10.0\,\mathrm{cm}$ that is forced to move at constant speed $v = 5.00\,\mathrm{m/s}$ along horizontal rails. The rod, rails, and connecting strip at the right form a conducting loop. The rod has resistance $0.400\,\Omega$; the rest of the loop has negligible resistance. A current $i = 100\,\mathrm{A}$ through the long straight wire at distance $a = 10.0\,\mathrm{mm}$ from the loop sets up a (nonuniform) magnetic field through the loop. Find the* ***(a)*** *emf and* ***(b)*** *current induced in the loop.* ***(c)*** *At what rate is thermal energy generated in the rod?* ***(d)*** *What is the magnitude of the force that must be applied to the rod to make it move at constant speed?* ***(e)*** *At what rate does this force do work on the rod?*




*Solution.* **(a)** Letting $x$ be the distance from the right end of the rails to the rod, we find an expression for the magnetic flux through the area enclosed by the rod and rails. By Ampere's Law, the field is $B = \mu_0 i/2\pi r$, where $r$ is the distance from the long straight wire. We consider an infinitesimal horizontal strip of length $x$ and width $dr$, parallel to the wire and a distance $r$ from it; it has area $A = xdr$, and the flux is

$$d\Phi_B = BdA = \frac{\mu_0 i}{2\pi r} xdr$$

The total flux through the area enclosed by the rod and rails is

$$\Phi_B = \frac{\mu_0 ix}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 ix}{2\pi}\ln\left(\frac{a+L}{a}\right)$$

According to Faraday's law the emf induced in the loop is

$$\begin{aligned}\varepsilon &= \frac{d\Phi_B}{dt} = \frac{\mu_0 i}{2\pi}\frac{dx}{dt}\ln\left(\frac{a+L}{a}\right) = \frac{\mu_0 iv}{2\pi}\ln\left(\frac{a+L}{a}\right)\\ &= \frac{(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(100\,\mathrm{A})(5.00\,\mathrm{m/s})}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right) = 2.40\times 10^{-4}\,\mathrm{V}\end{aligned}$$

**(b)** By Ohm's law, the induced current is

$$i_\ell = \varepsilon/R = (2.40\times 10^{-4}\,\mathrm{V})/(0.400\,\Omega) = 6.00\times 10^{-4}\,\mathrm{A}$$

Since the flux is increasing, the magnetic field produced by the induced current must be into the page in the region enclosed by the rod and rails. This means the current is clockwise.

**(c)** Thermal energy is being generated at the rate

$$P = i_\ell^2 R = (6.00 \times 10^{-4}\,\mathrm{A})^2(0.400\,\Omega) = 1.44 \times 10^{-7}\,\mathrm{W}$$

**(d)** Since the rod moves with constant velocity, the net force on it is zero. The force of the external agent must have the same magnitude as the magnetic force and must be in the opposite direction. The magnitude of the magnetic force on an infinitesimal segment of the rod, with length $dr$ at a distance $r$ from the long straight wire, is

$$dF_B = i_\ell B dr = (\mu_0 i_\ell i / 2\pi r) dr$$

We integrate to find the magnitude of the total magnetic force on the rod:

$$\begin{aligned} F_B &= \frac{\mu_0 i_\ell i}{2\pi} \int_a^{a+L} \frac{dr}{r} = \frac{\mu_0 i_\ell i}{2\pi} \ln\left(\frac{a+L}{a}\right) \\ &= \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(6.00 \times 10^{-4}\,\mathrm{A})(100\,\mathrm{A})}{2\pi} \ln\left(\frac{1.00\,\mathrm{cm} + 10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right) \\ &= 2.87 \times 10^{-8}\,\mathrm{N} \end{aligned}$$

Since the field is out of the page and the current in the rod is upward in the diagram, the force associated with the magnetic field is toward the right. The external agent must therefore apply a force of $2.87 \times 10^{-8}$ N, to the left.

**(e)** The external agent does work at the rate

$$P = Fv = (2.87 \times 10^{-8}\,\mathrm{N})(5.00\,\mathrm{m/s}) = 1.44 \times 10^{-7}\,\mathrm{W}$$

This is the same as the rate at which thermal energy is generated in the rod. All the energy supplied by the agent is converted to thermal energy. □

**Problem 6.** *A long solenoid has a diameter of* 12.0 cm. *When a current $i$ exists in its windings, a uniform magnetic field of magnitude* $B = 30.0\,\mathrm{mT}$ *is produced in its interior. By decreasing $i$, the field is caused to decrease at the rate of* $6.50\,\mathrm{mT/s}$. *Calculate the magnitude of the induced electric field* ***(a)*** 2.20 cm *and* ***(b)*** 8.20 cm *from the axis of the solenoid.*

*Solution.* Changing magnetic field induces an electric field. The induced electric field is given by $\oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi_B}{dt}$.

**(a)** The point at which we are evaluating the field is inside the solenoid, so

$$E(2\pi r) = -(\pi r^2)\frac{dB}{dt} \quad \Rightarrow \quad E = -\frac{1}{2}\frac{dB}{dt}r$$

The magnitude of the induced electric field is

$$|E| = \frac{1}{2}\frac{dB}{dt}r = \frac{1}{2}(6.5 \times 10^{-3}\,\mathrm{T/s})(0.0220\,\mathrm{m}) = 7.15 \times 10^{-5}\,\mathrm{V/m}$$

**(b)** Now the point at which we are evaluating the field is outside the solenoid, so

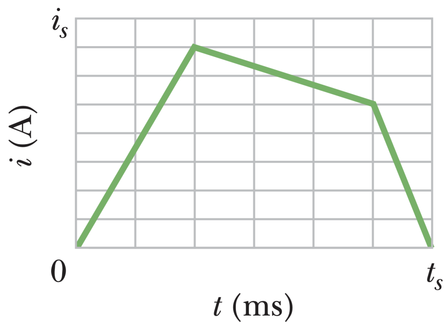
$$E(2\pi r) = -(\pi R^2)\frac{dB}{dt} \quad \Rightarrow \quad E = -\frac{1}{2}\frac{dB}{dt}\frac{R^2}{r}$$

The magnitude of the induced field is

$$|E| = \frac{1}{2}\frac{dB}{dt}\frac{R^2}{r} = \frac{1}{2}(6.5 \times 10^{-3}\,\mathrm{T/s})\frac{(0.0600\,\mathrm{m})^2}{0.0820\,\mathrm{m}} = 1.43 \times 10^{-4}\,\mathrm{V/m}$$

The magnitude of the induced electric field as a function of $r$ is shown to the right. Inside the solenoid, $r < R$, the field $|E|$ is linear in $r$. However, outside the solenoid, $r > R$, $|E| \sim 1/r$.
□

**Problem 7.** *The current $i$ through a 4.6 H inductor varies with time $t$ as shown by the graph, where the vertical axis scale is set by is $i_s = 8.0\,\mathrm{A}$ and the horizontal axis scale is set by $t_s = 6.0\,\mathrm{ms}$. The inductor has a resistance of $12\,\Omega$. Find the magnitude of the induced emf $\varepsilon$ during time intervals **(a)** 0 to 2 ms, **(b)** 2 ms to 5 ms, and **(c)** 5 ms to 6 ms. (Ignore the behaviour at the ends of the intervals.)*




*Solution.* During periods of time when the current varies linearly with time, the emf becomes $|\varepsilon| = L|\Delta i/\Delta t|$. For simplicity, we omit the absolute value signs in the following.

**(a)** For $0 < t < 2\,\mathrm{ms}$,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\mathrm{H})|7.0\,\mathrm{A} - 0|}{2.0 \times 10^{-3}\,\mathrm{s}} = 1.6 \times 10^4\,\mathrm{V}$$

**(b)** For $2\,\mathrm{ms} < t < 5\,\mathrm{ms}$,

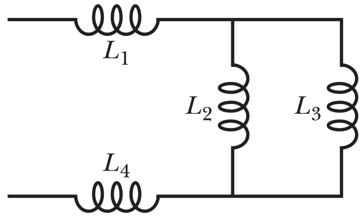
$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\mathrm{H})|5.0\,\mathrm{A} - 7.0\,\mathrm{A}|}{(5.0 - 2.0) \times 10^{-3}\,\mathrm{s}} = 3.1 \times 10^3\,\mathrm{V}$$

**(c)** For $5\,\mathrm{ms} < t < 6\,\mathrm{ms}$,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\mathrm{H})|0 - 5.0\,\mathrm{A}|}{(6.0 - 5.0) \times 10^{-3}\,\mathrm{s}} = 2.3 \times 10^4\,\mathrm{V}$$

□

**Problem 8.** *The inductor arrangement of the figure, with* $L_1 = 30.0\,\mathrm{mH}$*,* $L_2 = 50.0\,\mathrm{mH}$*,* $L_3 = 20.0\,\mathrm{mH}$*, and* $L_4 = 15.0\,\mathrm{mH}$*, is to be connected to a varying current source. What is the equivalent inductance of the arrangement?*




*Solution.* The equivalent resistance is

$$L_{eq} = L_1 + L_4 + L_{23} = L_1 + L_4 + \frac{L_2 L_3}{L_2 + L_3} = 30.0\,\mathrm{mH} + 15.0\,\mathrm{mH} + \frac{(50.0\,\mathrm{mH})(20.0\,\mathrm{mH})}{50.0\,\mathrm{mH} + 20.0\,\mathrm{mH}}$$
$$= 59.3\,\mathrm{mH}$$

□

**Problem 9.** *A solenoid that is* $85.0\,\text{cm}$ *long has a cross-sectional area of* $17.0\,\text{cm}^2$*. There are 950 turns of wire carrying a current of* $6.60\,\text{A}$*.* ***(a)*** *Calculate the energy density of the magnetic field inside the solenoid.* ***(b)*** *Find the total energy stored in the magnetic field there (neglect end effects).*

*Solution.* The magnetic energy density is given by $u_B = B^2/2\mu_0$, where $B$ is the magnitude of the magnetic field at that point. Inside a solenoid, the magnitude of the magnetic field is $B = \mu_0 n i$, where

$$n = (950\,\text{turns})/(0.850\,\text{m}) = 1.118 \times 10^3\,\text{m}^{-1}$$

Thus, the energy density is

$$u_B = \frac{B^2}{2\mu_0} = \frac{(\mu_0 n i)^2}{2\mu_0} = \frac{1}{2}\mu_0 n^2 i^2$$

Since the magnetic field is uniform inside an ideal solenoid, the total energy stored in the field is $U_B = u_B V$, where $V$ is the volume of the solenoid.

**(a)** Substituting the values given, we find the magnetic energy density to be

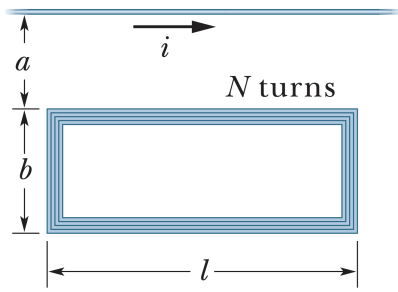
$$u_B = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(1.118 \times 10^3\,\text{m}^{-1})^2(6.60\,\text{A})^2 = 34.2\,\text{J/m}^3$$

**(b)** The volume $V$ is calculated as the product of the cross-sectional area and the length.

$$U_B = (34.2\,\text{J/m}^3)(17.0 \times 10^{-4}\,\text{m}^2)(0.850\,\text{m}) = 4.94 \times 10^{-2}\,\text{J}$$

Note the similarity between $u_B = \dfrac{B^2}{2\mu_0}$, the energy density at a point in a magnetic field, and $u_E = \dfrac{1}{2}\varepsilon_0 E^2$, the energy density at a point in an electric field. Both quantities are proportional to the square of the fields. □

**Problem 10.** *A rectangular loop of $N$ closely packed turns is positioned near a long straight wire as shown in the figure. What is the mutual inductance $M$ for the loop-wire combination if $N = 100$, $a = 1.0\,\text{cm}$, $b = 8.0\,\text{cm}$, and $l = 30\,\text{cm}$?*




*Solution.* The flux over the loop cross-section due to the current $i$ in the wire is given by

$$\Phi = \int_a^{a+b} B_{\text{wire}} l\, dr = \int_a^{a+b} \frac{\mu_0 i l}{2\pi r} dr = \frac{\mu_0 i l}{2\pi} \ln\left(1 + \frac{b}{a}\right)$$

Thus,

$$M = \frac{N\Phi}{i} = \frac{N\mu_0 l}{2\pi} \ln\left(1 + \frac{b}{a}\right)$$

From the formula for $M$ obtained above, we have

$$M = \frac{(100)(4\pi \times 10^{-7}\,\text{H/m})(0.30\,\text{m})}{2\pi} \ln\left(1 + \frac{8.0}{1.0}\right) = 1.3 \times 10^{-5}\,\text{H}$$

□